Conference

Structures in Banach Spaces

Erwin Schrödinger Institute

Vienna, 17 – 21 March 2025

Book of Abstracts

Invited speakers

- **Piotr Borodulin-Nadzieja**, University of Wrocław
- **Marián Fabian**, Czech Academy of Sciences, Prague
- **María Japón**, University of Seville
- **Piotr Koszmider**, Polish Academy of Sciences
- **Niels Laustsen**, Lancaster University
- **Jordi López-Abad**, UNED
- **Witold Marciszewski**, University of Warsaw
- **Eva Pernecká**, Czech Technical University
- **Abraham Rueda Zoca**, University of Granada
- **Stevo Todorčević**, Université de Paris/University of Toronto
- **Alessandro Vignati**, Université Paris Cité

Scientific committee

- **Antonio Avilés**, University of Murcia
- **Vera Fischer**, University of Vienna
- **Grzegorz Plebanek**, University of Wrocław
- **Damian Sobota**, University of Vienna

Organizing committee

- **The Erwin Schrödinger Institute**, Vienna
- **Vera Fischer**, University of Vienna
- **Damian Sobota**, University of Vienna
- **Tomasz Żuchowski**, University of Vienna/University of Wrocław

Detailed schedule

Monday 17.03

09:00 – 09:30	*Registration & Welcome*
09:30 – 10:15	**Marián Fabian**, *Asplund and weak Asplund properties of Banach spaces and beyond*
10:30 – 11:15	**Stevo Todorčević**, *Compact sets of Baire class one functions*
11:15 – 11:45	*Coffee Break*
11:45 – 12:05	**José Ansorena**, *Uniqueness of unconditional basis in Banach spaces: overview and recent advances*
12:15 – 12:35	**Olav Nygaard Dovland**, *Diameter 2 properties, a survey*
12:35 – 14:45	*Lunch Break*
14:45 – 15:30	**María Japón**, *Some applications of Boolean algebras and the Stone representation theorem to metric fixed point theory*
15:30 – 16:00	*Coffee Break*
16:00 – 16:20	**Christian Bargetz**, *On extremal nonexpansive mappings*
16:25 – 16:45	**Sebastián Tapia García**, *Horofunction extension of metric spaces and Banach spaces*

Tuesday 18.03

09:00 – 09:45	**Alessandro Vignati**, *The weak Extension Principle and its coarse version*
10:00 – 10:45	**Niels Laustsen**, *The Baernstein and Schreier spaces, and operators on them*
10:45 – 11:15	*Coffee Break*
11:15 – 11:35	**Henrik Wirzenius**, *Closed subideals of bounded operators*
11:45 – 12:05	**Anna Pelczar-Barwacz**, *A criterion for* $2^{\mathfrak{c}}$ *operator ideals on Banach spaces*
12:15 – 12:35	**Miguel Ángel Ruiz Risueño**, *Operators with dense range and in Banach spaces with a separable quotient*
12:35 – 12:45	*Group Photo*
12:45 – 14:45	*Lunch Break*
14:45 – 15:30	**Piotr Koszmider**, *On subspaces of indecomposable Banach spaces*
15:30 – 16:00	*Coffee Break*
16:00 – 16:20	**Wiesław Kubiś**, *Generic operators*

16:25 – 16:45	**Thomas Speckhofer**, *Dimension dependence of factorization problems*

Wednesday 19.03

09:00 – 09:45	**Abraham Rueda Zoca**, *Space of vector valued Lipschitz functions and the Daugavet property*
10:00 – 10:45	**Eva Pernecká**, *De Leeuw representation of functionals in Lipschitz-free spaces*
10:45 – 11:15	*Coffee Break*
11:15 – 11:35	**Juan Guerrero Viu**, *Projective tensor products: The pursuit of norm-attainment*
11:45 – 12:05	**Jerzy Kąkol**, *The Grothendieck property for spaces* $\mathrm{Lip}_0(M)$ *of Lipschitz functions*
12:15 – 12:35	**José Orihuela**, *An old problem of Joram Lindenstrauss*
12:35 – 14:45	*Lunch Break*
14:45 – 15:05	**Jesús M. F. Castillo**, *Somehow off-limits: The 3-space problem for Isomorphic Polyhedrality*
15:15 – 15:35	**Tommaso Russo**, *Discrete subgroups of Banach spaces and lattice tilings*
15:45 – 16:05	**Miguel García Bravo**, *Density of smooth function with no critical points within the set of continuous functions on Banach spaces*
16:10 – 16:30	**Daylen Thimm**, *True* σ*-porosity for alternating projection orders*
19:00	*Conference Dinner, Restaurant Heuriger Mayer am Pfarrplatz*

Thursday 20.03

09:00 – 09:45	**Jordi López-Abad**, F_σ*-ideals, colorings, and representation in Banach spaces*
10:00 – 10:45	**Piotr Borodulin-Nadzieja**, *Combinatorial Banach spaces*
10:45 – 11:15	*Coffee Break*
11:15 – 11:35	**Sebastian Jachimek**, *On the spaces dual to combinatorial Banach spaces*
11:45 – 12:05	**Víctor Olmos Prieto**, *The Banach-Saks rank of a separable weakly compact set*
12:15 – 12:35	**Zdeněk Silber**, *A separable Banach space of nontrivial Baire order*
12:35 – 14:45	*Lunch Break*
14:45 – 15:30	**Witold Marciszewski**, *Counting Banach spaces C(K)*
15:30 – 16:00	*Coffee Break*

16:00 – 16:20	**Maciej Korpalski**, *Semadeni derivative of Banach spaces and functions on nonmetrizable rectangles*
16:25 – 16:45	**Eugene Bilokopytov**, *Von Neumann–Maharam problem for vector lattices*

Friday 21.03

09:00 – 09:20	**Arkady Leiderman**, *On w^*-binormality of the dual space $C_k(X)'$*
09:30 – 09:50	**Todor Manev**, *Continuous functions on Fedorchuk compacta*
10:00 – 10:20	**Ondřej Kalenda**, *Simpliciality of vector-valued function spaces*
10:20 – 10:50	*Coffee Break*
10:50 – 11:10	**David Muñoz-Lahoz**, *AM-algebras*
11:20 – 11:40	**Luis David Reyes Sáenz**, *Q-points, Q-measures, and L-orthogonality*
11:50 – 12:10	**Alberto Salguero-Alarcón**, *Free objects in p-Banach lattices*
12:15 – 12:35	**Ondřej Zindulka**, *Fractal measures in Polish groups and Banach spaces: cardinal invariants*
12:35 – 12:45	*Closing*

MONDAY 17.03

09:30 – 10:15, Plenary talk

Marián Fabian

Czech Academy of Sciences, Prague

Asplund and weak Asplund properties of Banach spaces and beyond

First, we review the classical theory of Asplund and weak Asplund properties of Banach spaces. Equivalent, sufficient, and necessary conditions for them are listed. In particular, the role of differentiability of the norm is shown. Stability under various gymnastics is considered. Open questions naturally raised are mentioned. Deeper structural results for Asplund property are shown via projectional resolutions of identity (PRI) and a recent mutant of it, projectional skeletons. Here, the so-called rich families of separable subspaces arise naturally. An application in (sub-)differentiability theory is sketched then. Finally, we raise a question whether it is possible to study both weak Asplund and Asplund properties in locally convex spaces. Nowadays, this is a quite vibrant area. Sometimes we can imitate the technology from Banach spaces, but not always. Some of the obtained results are a bit surprising. Definitely, it makes sense to go beyond Banach spaces. Just to get a taste: Mazur's theorem that separable Banach spaces are weak Asplund can be extended to separable Baire t.l.s.; the non-Banach spaces $C_k(\mathbb{Q})$ and $\mathbb{R}^{\aleph}$, with $\aleph$ any cardinal, are Asplund; and given any Banach space X, then (X, w) and (X^*, w^*) are Asplund.

10:30 – 11:15, Plenary talk

Stevo Todorčević

Université de Paris/University of Toronto

Compact sets of Baire class one functions

Since the work of Haskell Rosenthal from the 1970's it has become apparent that structure theory of compact sets of Baire class one functions on Polish (or more generally on K-analytic) spaces can interpret important problem from the geometry of Banach spaces. This will be an overview of this area starting from the classical theory and ending with some some more recent results.

14:45 – 15:30, Plenary talk

María Japón

University of Seville

Some applications of Boolean algebras and the Stone representation theorem to metric fixed point theory

A metric space (B, d) is said to have the fixed point property (FPP) if every 1-Lipschitz operator $T : B \to B$ has a fixed point.

During this talk, our metric space will be the closed unit ball of a Banach space of continuous functions $C(K)$, for K a Hausdorff compact topological space. We are interested in identifying topological properties of the compact set K that are connected to the failure or to the fulfillment of the FPP for the closed unit ball B of $C(K)$.

This question arises after observing that the two opposite behaviours hold. For instance, if K is the one-point compactification of $\mathbb{N}$, it is easy to check that the closed unit ball of $C(K)$ fails the FPP. In contrast, if $K = \beta\mathbb{N}$, the Stone–Čech compactification of $\mathbb{N}$, the closed unit ball does verify the FPP (extremally disconnection, hyperconvexity and injectivity play their role here).

Firstly, we will prove that K being a topological F-space is a necessary condition for the FPP, which will allow us to dismiss the FPP for different classes of compact spaces.

We will next focus on some particular examples of topological F-spaces, in particular, on the remainder of $\beta\mathbb{N}$, that is, $\mathbb{N}^* = \beta\mathbb{N} \setminus \mathbb{N}$ formed by all free ultrafilters.

While $C(\beta\mathbb{N})$ is isometric to ℓ_∞, $C(\mathbb{N}^*)$ is isometric to the quotient Banach space ℓ_∞/c_0. A natural question arises: Does the closed unit ball of $C(\mathbb{N}^*)$ or, equivalently, the closed unit ball of ℓ_∞/c_0, have the FPP?

We will prove that Boolean algebras, the Stone representation theorem and Boolean retractions will provide us with an answer to the previous question, under the umbrella of the continuum hypothesis.

Some open problems will be exposed.

The results included in this talk will be eventually published in a joint paper together with Antonio Avilés (Murcia University), Christopher Lennard (Pittsburgh University), Gonzalo Martinez-Fernandez (Murcia University) and Adam Stawski (Pittsburgh University)

11:45 – 12:05

José Ansorena

Universidad de La Rioja, Logroño

Uniqueness of unconditional basis in Banach spaces: overview and recent advances

Determining the existence and uniqueness of an unconditional basis is a relevant topic within the study of lattice structures in separable Banach (and quasi-Banach) spaces. In this talk, we will review the main results in this line of research from the first theorems obtained in the 1930s to the recent advances achieved by the authors.

12:15 – 12:35

Olav Nygaard Dovland

University of Agder

Diameter 2 properties, a survey

The talk will be a survey over various diameter 2 properties, together with some history of how it all evolved after starting just around the year 2000. How rotund diameter 2 spaces can be is a question of special interest. There are also some natural open questions that will be presented. An example of such a question goes like this: M-embedded spaces have the property that all subspaces have very strong diameter 2 properties. Can we characterize the Banach spaces where all subspaces have the diameter 2 property?

16:00 – 16:20

Christian Bargetz

University of Innsbruck

On extremal nonexpansive mappings

A nonexpansive self-mapping of a bounded, closed and convex subset of a Banach space is called extremal if it does not have a representation as a non-trivial convex combination of nonexpansive mappings. We show that on the unit ball of many classical Banach spaces surjective isometries are extremal among nonexpansive mappings. We also show that the typical, in the sense of Baire category, nonexpansive mapping is close to be extremal. This is joint work with Katriin Pirk and Michael Dymond.

16:25 – 16:45

Sebastián Tapia García

VADOR, TU Wien

Horofunction extension of metric spaces and Banach spaces

In this talk we provide a necessary and sufficient condition for the horofunction extension of a metric space to be a compactification. The condition clarifies previous results on proper metric spaces and geodesic spaces and yields the following characterization: a Banach space is Gromov-compactifiable under any renorming if and only if it does not contain an isomorphic copy of ℓ_1. In addition, it is shown that, up to an adequate renorming, every Banach space is Gromov-compactifiable. Therefore, the property of being Gromov-compactifiable is not invariant under bi-Lipschitz equivalence.

This is a joint work with A. Daniilidis, M.I. Garrido and J. Jaramillo.

TUESDAY 18.03

9:00 – 9:45, Plenary talk

Alessandro Vignati

Université Paris Cité

The weak Extension Principle and its coarse version

In the late 90s Farah formulated the weak Extension Principle (wEP) for Čech–Stone remainders of zero-dimensional topological spaces. In a nutshell, the wEP asks to completely understand continuous functions between such Čech–Stone remainders. While this principle does not follow from the usual ZFC Axioms, being false for example under the Continuum Hypothesis, Farah showed some instances of wEP are a consequence of fairly mild axioms such as the Open Colouring Axiom and Martin's Axiom.

We explore generalisations of the wEP to all Čech–Stone remainders of locally compact metrizable spaces, and state a coarse geometric version of wEP allowing to obtain (again under reasonable set theoretic assumptions) rigidity results for Higson coronas, boundary spaces arising in coarse geometry.

10:00 – 10:45, Plenary talk

Niels Laustsen

Lancaster University

The Baernstein and Schreier spaces, and operators on them

I shall report on recent joint work with James Smith (https://doi.org/10.1016/j.jmaa.2025.129235) in which we study the lattice of closed ideals of bounded operators on two families of Banach spaces: the Baernstein spaces B_p for $1 < p < \infty$ and the p-convexified Schreier spaces S_p for $1 \leq p < \infty$. Our main conclusion is that there are $2^{\mathfrak{c}}$ many closed ideals that lie between the ideals of compact and strictly singular operators on each of these spaces, and also $2^{\mathfrak{c}}$ many closed ideals that contain projections of infinite rank.

Counterparts of results of Gasparis and Leung using a numerical index to distinguish the isomorphism types of subspaces spanned by subsequences of the unit vector basis for the classical Schreier space S_1 play a key role in the proofs, as does the Johnson–Schechtman technique for constructing $2^{\mathfrak{c}}$ many closed ideals of operators on a Banach space.

I intend to begin from first principles, without assuming any prior familiarity with the Schreier sets or the associated Banach spaces mentioned in the title, with emphasis on the combinatorial aspects of our work, notably through the above-mentioned index of Gasparis and Leung.

14:45 – 15:30, Plenary talk

Piotr Koszmider

Polish Academy of Sciences, Warsaw

On subspaces of indecomposable Banach spaces

We investigate the class of Banach spaces that can be subspaces of indecomposable Banach spaces of densities up to continuum showing that it includes all Banach spaces of such densities which do not admit ℓ_∞ as quotients. It remains open if every Banach space that does not contain a copy of ℓ_∞ is a subspace of an indecomposable Banach space. The constructed indecomposable Banach space including an appropriate Banach space X is a subalgebra of the algebra $C(K)$, where K is the Čech–Stone remainder of the cartesian product of the dual ball of X and the euclidean plane. The results were obtained together with Zdeněk Silber.

11:15 – 11:35

Henrik Wirzenius

Czech Academy of Sciences, Prague

Closed subideals of bounded operators

A closed subalgebra $J \subset L(X)$ of the bounded operators on a Banach space X is called a closed subideal of $L(X)$ if there is a closed ideal I of $L(X)$ such that J is a closed ideal of I. The subideal J is called non-trivial if it is not an ideal of $L(X)$.

More generally, we call J a closed n-subideal of $L(X)$ if there are closed subalgebras $J_0, \ldots, J_n$ of $L(X)$ such that $J = J_n \subset \ldots \subset J_1 \subset J_0 = L(X)$ and each J_k is a closed ideal of J_{k-1}.

In this talk I will describe examples and properties of non-trivial closed subideals and closed n-subideals of $L(X)$ for various Banach spaces X. The talk is based on a joint ongoing work with Hans-Olav Tylli (Helsinki).

11:45 – 12:05

Anna Pelczar-Barwacz

Jagiellonian University, Kraków

A criterion for $2^{\mathfrak{c}}$ operator ideals on Banach spaces

We present a general criterion based on the asymptotic behavior of basic sequences and Johnson–Schechtman technique, which guarantees large cardinality of the lattice of closed operator ideals in the algebra of bounded operators on a Banach space. The method yields $2^{\mathfrak{c}}$ closed operator ideals on a class of Lorentz sequence spaces, combinatorial spaces defined by compact families of finite subsets of integers, and spaces built on their basis — their p-convex versions and Baernstein spaces (extending the results of R.M. Causey – APB, N.J. Laustsen – J.Smith, A.Manoussakis – APB) and provides another approach to case of Rosenthal spaces, solved by W.B.Johnson and G. Schechtman.

12:15 – 12:35

Miguel Ángel Ruiz Risueño

Universidad de Castilla-La Mancha

Operators with dense range and in Banach spaces with a separable quotient

Given a Banach space E with a separable infinite-dimensional quotient, a bounded linear dense-range operator T from E into another Banach space F, and proper dense range R in E, we provide a condition to ensure the existence of a closed subspace E_1 such that E/E_1 is infinite-codimensional and separable, $R + E_1$ is infinite-codimensional, and $T(E_1)$ is dense in F. As an application of this result, we deduce that if X and Y are proper quasicomplements in a Banach space E, and X has a separable quotient, then X contains a closed subspace X_1 such that $\dim(X/X_1) = \infty$ and $X_1 + Y$ is dense in E. This fact generalizes a theorem of Johnson, which ensures that the assertions hold true whenever the subspace X is WCG. Some refinements of these results in the case that E has weak*-separable dual are also given, which extend some of those obtained by Fonf, Lajara, Troyanski and Zanco in [1].

References

[1] V. P. Fonf, S. Lajara, S. Troyanski and C. Zanco, *Operator ranges and quasicomplemented subspaces of Banach spaces*, Studia Math. **246** (2) (2019), 203–216.

16:00 – 16:20

Wiesław Kubiś

Czech Academy of Sciences, Prague

Generic operators

We present a general method for constructing generic morphisms. Here, “generic” may be understood as either being the most complicated one (in a suitable class of morphisms) or having a residual orbit with respect to the natural action of the autormorphism group of its domain. Nevertheless, we shall give a precise and general definition through a natural abstract Banach–Mazur game.

In certain categories of Banach spaces the above method brings generic non-expansive linear operators. Some of them are universal in the sense that they capture all non-expansive operators with a given range. For example, a universal generic operator exists on the Gurarii space (a result from 2015, joint with J. Garbulińska-Węgrzyn).

As a concrete new application, we show the Kadec–Pełczyński–Wojtaszczyk universal space admits a universal generic operator.

16:25 – 16:45

Thomas Speckhofer

Johannes Kepler University, Linz

Dimension dependence of factorization problems

For a natural number n, let Y_n denote the linear span of the first $n+1$ levels of the Haar system in a Haar system Hardy space (i.e., in a rearrangement-invariant function space or a related space such as dyadic H^1). Consider the following question: Given n and $\delta > 0$, how large does N have to be chosen such that for every linear operator T on Y_N with norm at most 1 and with δ-large positive diagonal entries (with respect to the Haar basis), there exists a factorization of the identity $I_{Y_n} = ATB$, where A and B satisfy a uniform upper bound such as $\|A\|\|B\| \leq 2$? This problem is closely related to the Restricted Invertibility Theorem by J. Bourgain and L. Tzafriri. We show that in general, an inequality of the form $N \geq Cn^2$ is sufficient, whereas if the Haar system is unconditional in the underlying space, then $N \geq Cn$ suffices. This amounts to a quasi-polynomial or, respectively, polynomial dependence between the dimensions of Y_N and Y_n.

WEDNESDAY 19.03

9:00 – 9:45, Plenary talk

Abraham Rueda Zoca

University of Granada

Space of vector valued Lipschitz functions and the Daugavet property

Recall that a Banach space X has the *Daugavet property* if, given any rank-one linear and continuous operator $T : X \longrightarrow X$ it follows that

$$\|T + \mathrm{I}\| = 1 + \|T\|,$$

where $\mathrm{I} : X \longrightarrow X$ stands for the identity operator. In the context of the Lipschitz free space over a complete metric space, it is known that $\mathcal{F}(M)$ has the Daugavet property if and only if the metric space M is *length*, i.e., if for every pair of distinct points $x, y \in M$ $d(x, y)$ equals the inf. of the length of all the rectifiable curves joining x and y ([1]), and if and only if its dual $\mathrm{Lip}(M)$ has the Daugavet property. In this talk we will analyze the question when $\mathrm{Lip}(M, X)$ has the Daugavet property for every Banach space X. The results of this talk are part of the preprint [2] in collaboration with R. Medina.

References

[1] L. García-Lirola, A. Procházka and A. Rueda Zoca, A characterisation of the Daugavet property in spaces of Lipschitz functions, J. Math. Anal. Appl. 464 (2018), 473–492.

[2] R. Medina and A. Rueda Zoca, A characterisation of the Daugavet property in spaces of vector-valued Lipschitz functions, preprint. Available at ArXiV.org with reference arXiv:2305.05956.

10:00 – 10:45, Plenary talk

Eva Pernecká

Czech Technical University, Prague

De Leeuw representation of functionals in Lipschitz-free spaces

At present, no general representation theorem for duals of spaces of Lipschitz functions over a pointed metric space is known. A useful substitute is provided by a construction due to K. de Leeuw from the 1960s, which allows us to view functionals on spaces of Lipschitz functions as Radon measures integrating the incremental quotients of functions. The main drawback of such a representation is that the representing measures are not unique.

We will focus on functionals from the canonical predual of the space of Lipschitz functions, the Lipschitz-free space. We will discuss the existence of "nice" representing measures for such functionals and present some applications to the isometric theory of Lipschitz-free spaces. In particular, we will show that a Choquet-like theory for De Leeuw representations, recently developed by R. J. Smith, leads to an "inner regularity" result for elements of Lipschitz-free spaces and the characterisation of the extreme points of their unit balls.

The talk will be based on joint work with Ramón J. Aliaga (Universitat Politècnica de València) and Richard J. Smith (University College Dublin).

11:15 – 11:35

Juan Guerrero Viu

University of Zaragoza

Projective tensor products: The pursuit of norm-attainment

Given X, Y two Banach spaces, we can consider the projective tensor product $X\widehat{\otimes}_\pi Y$ which is used to linearize bilinear functions. In this talk we analyse when every element of $X\widehat{\otimes}_\pi Y$ attains its projective norm. We prove that this is the case if X is the dual of a subspace of a predual of an $\ell_1(I)$ space and Y is 1-complemented in its bidual under approximation properties assumptions. This result allows us to provide some new examples where X is a Lipschitz-free space. This work has been carried out in collaboration with Luis C. García-Lirola (Univesity of Zaragoza) and Abraham Rueda Zoca (University of Granada).

11:45 – 12:05

Jerzy Kąkol

Adam Mickiewicz University, Poznań

The Grothendieck property for spaces $\mathrm{Lip}_0(M)$ of Lipschitz functions

We study the following

Problem. *For which metric spaces M, in particular for which Banach spaces M, do the spaces* $\mathrm{Lip}_0(M)$ *have the Grothendieck property?*

Recall that a Banach space E is called *Grothendieck* if every weak* convergent sequence in the dual space E^* converges weakly. Typical examples of Grothendieck spaces are reflexive spaces, the space ℓ_∞ or more generally spaces $C(K)$ for K extremally disconnected, the space H^∞ of all bounded analytic functions on the unit disk and von Neumann algebras. It seems that apart from $\mathrm{Lip}_0([0,1]) \simeq \ell_\infty$ there is no known example of a Banach space $\mathrm{Lip}_0(M)$ which is a Grothendieck space. We provide a number of conditions for metric spaces M implying that the corresponding spaces $\mathrm{Lip}_0(M)$ are not Grothendieck. For example, if a Banach space E is a $C(K)$-space, $L_1(\mu)$-space, $\mathrm{Lip}_0(M)$-space, or $\mathcal{F}(M)$-space, then $\mathrm{Lip}_0(E)$ is not Grothendieck.

The presented results are based on a joint work with Christian Bargetz and Damian Sobota.

12:15 – 12:35

José Orihuela

University of Murcia

An old problem of Joram Lindenstrauss

Following Lindenstrauss [1] we read:

Question: Characterize those Banach spaces which have an equivalent strictly convex norm. It is easily verified that every separable Banach space has an strictly convex norm. The same is true for a general WCG space. On the other hand, it was shown by Day that there exist Banach spaces which do not have an equivalent strictly convex norm. Some conjectures concerning a possible answer to the question were shown to be false by Dashiell and Lindenstrauss. This results shows that even for C(K) spaces it seems to be a delicate and presumably difficult question to decide under which condition there exists an equivalent strictly convex norm.

Our answer is that a Banach space admits an equivalent rotund norm if, and only if, it has another one with separable faces. It follows that every compact space K where the separable subsets are metrizable, i.e. the so called monolithic spaces, the Banach space $C(K)$ has an equivalent strictly convex norm. Even more, when every countable subset of a fixed dense subset of K is going to be included in a separable retract of K we also have an equivalent strictly convex norm on $C(K)$, thus our results include the class of Valdivia compact spaces. Moreover, we will show in this talk a link between Fréchet norms and convex renormings.

Joint work with Vicente Montesinos [2].

References

[1] J. Lindenstrauss. Some open problems in Banach space theory. *Séminaire Choquet. Initiation à l' analyse tome 15, 1975-1976, Exp. No. 18.* Pages 1–9. Secrétariat mathématique, Paris, 1975-76.

[2] V. Montesinos and J. Orihuela. *Weak compactness and separability of faces for convex renormings of Banach spaces.* Preprint 2024.

14:45 – 15:05

Jesús M. F. Castillo

University of Extremadura, Imuex

Somehow off-limits: The 3-space problem for Isomorphic Polyhedrality

A Banach space is *polyhedral* if every finite dimensional subspace is isometrically isomorphic to a subspace of some finite-dimensional ℓ_∞^n. Banach spaces admitting a polyhedral renorming are called *isomorphically polyhedral.* Many times, and in many places [2, 3], has the author asked whether to be isomorphically polyhedral is a 3-space property: which means whether it is true that whenever a subspace Y of a Banach space Z as well as the corresponding quotient $X = Z/Y$ are isomorphically polyhedral then Z has to be isomorphically polyhedral. Equivalently, if every twisted sum of two isomorphically polyhedral spaces is isomorphically polyhedral. In this talk we plan to pinpoint the difficulties (we can see) to solve this problem and present the available relevant partial results.

References

[1] F. Cabello Sánchez, J.M.F. Castillo, *Homological methods in Banach space theory*, Cambridge Studies in Advanced Math. 203, (2023) Cambridge Univ. Press. ISBN. 9781108778312.

[2] J.M.F. Castillo, P.L. Papini, *Hepheastus account on Trojanski's polyhedral war*, Extracta Math. Vol. 29, (2014) 35 – 51.

[3] J.M.F. Castillo, P.L. Papini, *On isomorphically polyhedral $\mathscr{L}_\infty$-spaces*, J. Funct. Anal. 270 (2016) 2336–2342.

[4] J.M.F. Castillo, A. Salguero, *Twisted sums of $c_0(I)$*, Quaestiones Math. 46 (2023) 2339-2354.

[5] J.M.F. Castillo, A. Salguero, *Polyhedrality for twisted sums with $C(\omega^\alpha)$*, preprint 2024.

[6] V.P. Fonf, *On the boundary of a polyhedral Banach space*, Extracta Math. 15 (2000) 145–154.

[7] V.P. Fonf, A.J. Pallares, R.J. Smith, S. Troyanski, *Polyhedral norms on non-separable Banach spaces*, J. Funct. Anal. 255 (2008) 449–470.

[8] L. Veselý, *Boundary of polyhedral spaces: An alternative proof*, Extracta Math. 15 (2000) 213–218.

15:15 – 15:35

Tommaso Russo

University of Innsbruck

Discrete subgroups of Banach spaces and lattice tilings

In 2008 Dilworth, Odell, Schlumprecht, and Zsák proved that every separable Banach space contains a net that is an (additive) subgroup. In this talk we will explain how a modification of an argument due to Victor Klee permits to obtain a shorter self-contained proof of the said result and also to extend the result to non-separable Banach spaces. In the second part of the talk, we focus on the non-separable Hilbert space $\ell_2(\Gamma)$, for $|\Gamma| = \mathfrak{c}$, and show that it contains a subgroup which is $(\sqrt{2}+)$-separated and 1-dense. We also explain how this construction answers a problem originating from the theory of tilings. Joint work with Carlo Alberto De Bernardi and Jacopo Somaglia.

15:45 – 16:05

Miguel García Bravo

Complutense University of Madrid

Density of smooth function with no critical points within the set of continuous functions on Banach spaces

In 1969, Kupa proved that there exists functions $f : \ell_2(\mathbb{N}) \to \mathbb{R}$ of class C^∞ so that $f(C_f) = [0,1]$, where $C_f = \{x \in \ell_2 : Df(x) = 0\}$ is the set of critical points. This shows the failure of the classical Morse–Sard theorem in infinite dimensions. However, in 2004, Azagra and Cepedello proved that any continuous function $f : \ell_2(\mathbb{N}) \to \mathbb{R}$ can be uniformly approximated by C^∞ functions without critical points. This type of result can be seen as an approximate version of the Morse–Sard theorem.

Several authors have studied to what extend Azagra and Cepedello's result can be generalized to functions $f : E \to F$ where E and F are Banach spaces with $\dim(E) = \infty$. In 2019, Azagra, Dobrowolski and myself considered the case of separable Banach spaces and showed the same type of result to be true in many situations. The main purpose of this talk is to discuss the nonseparable case, which has been treated much more recently. In particular, in this presentation we will show an approximate Morse–Sard type result for the case of continuous functions $f : \ell_2(\Gamma) \to \mathbb{R}$, where Γ is an arbitrary infinite set. Moreover, similar results are also true for more general nonseparable Banach spaces E and F.

This is a joint work with Daniel Azagra and Mar Jiménez-Sevilla.

16:10 – 16:30

Daylen Thimm

University of Innsbruck

True σ-porosity for alternating projection orders

Consider three closed linear subspaces C_1, C_2, and C_3 of a Hilbert space H and the orthogonal projections P_1, P_2 and P_3 to them. Halperin showed that a point in $C_1 \cap C_2 \cap C_3$ can be found by iteratively projecting any point $x_0 \in H$ onto all the sets in a periodic fashion. The limit point is then the projection of x_0 onto $C_1 \cap C_2 \cap C_3$. Nevertheless, a non-periodic projection order may lead to a non-convergent projection series, as shown by Kopecká, Müller, and Paszkiewicz. This raises the question how many projection orders in $\{1, 2, 3\}^{\mathbb{N}}$ are "well behaved" in the sense that they lead to a convergent projection series. De Brito, Melo, and da Cruz Neto showed that the "well behaved" projection orders form a large subset in the sense of measure, as they have full product measure. We show that also from a topological viewpoint the set of "well behaved" projection orders is a large subset: it contains a co-σ-porous subset with respect to a metric inducing the product topology.

THURSDAY 20.03

9:00 – 9:45, Plenary talk

Jordi López-Abad

UNED

F_σ-ideals, colorings, and representation in Banach spaces

We study B- and C-ideals associated with sequences in Banach spaces, where $C((x_n)n)$ consists of sets for which the series $\sum_{n\in A} x_n$ is unconditionally convergent, and $B((x_n)_n)$ consists of sets where the series is weakly unconditionally convergent. We describe these ideals in universal function spaces, particularly in $C([0,1])$ and $C(2^{\mathbb{N}})$, addressing a question by Borodulin-Nadzieja et al.

A key aspect is the role of c_0-saturated spaces and their connection to c-coloring ideals, which exhibit a rich combinatorial structure. We show that for $d \geq 3$, the random d-homogeneous ideal is pathological, construct hereditarily non-pathological c-coloring ideals, and prove that every B-ideal in $C(K)$, for countable K, contains a c-coloring ideal.

These results highlight the interplay between combinatorial properties of ideals and their Banach space representations. This is a joint work with V. Olmos and C. Uzcátegui.

10:00 – 10:45, Plenary talk

Piotr Borodulin-Nadzieja

University of Wrocław

Combinatorial Banach spaces

I will present some theory of Banach spaces induced by graphs and, more generally, hypergraphs (in the same sense as the Schreier family induces the Schreier space). I will show some structural theorems and a couple of old and new examples, indicating resemblances with the theory of analytic P-ideals.

14:45 – 15:30, Plenary talk

Witold Marciszewski

University of Warsaw

Counting Banach spaces $C(K)$

I will present some results concerning the following general problems:
Let $\mathcal{K}$ be a class of compact spaces.

1. How to classify (up to isomorphisms) Banach spaces $C(K)$ of real-valued continuous functions on K (with supremum norm), for $K \in \mathcal{K}$?
2. How many isomorphic types of $C(K)$ are there, for $K \in \mathcal{K}$?

The classical result of Bessaga and Pełczyński gives us a complete classification of $C(K)$, for the class of countable compact spaces K; in particular, we have ω_1 isomorphic types of such spaces $C(K)$. On the other hand, Milutin's theorem says that, for the class of uncountable metrizable compact spaces K, we have only one isomorphic type of spaces $C(K)$.

I will discuss two well-known classes of compact spaces of weight ω_1, for which problem 2 is not decidable in ZFC.

The first of these classes is the class $\mathcal{K}$ of compact spaces generated by families of almost disjoint subsets of the set of natural numbers ω, usually associated with the names of Mrówka, Isbell, Franklin, or Aleksandrov and Urysohn. Assuming the continuum hypothesis, we have 2^{2^ω} isomorphic types of $C(K)$, for $K \in \mathcal{K}$. In turn, assuming Martin's axiom and negation of the continuum hypothesis, for all $K, L \in \mathcal{K}$ with $w(K) = w(L) = \omega_1$, the spaces $C(K)$ and $C(L)$ are isomorphic (joint results with R. Pol, F. Cabello Sánchez, J. Castillo, G. Plebanek, A. Salguero-Alarcón).

The second class considered is the class $\mathcal{L}$ of separable, compact linearly ordered spaces of weight ω_1. Again, assuming the continuum hypothesis, we have 2^{2^ω} isomorphic types of $C(K)$, for $K \in \mathcal{L}$. On the other hand, assuming a certain axiom proposed by Baumgartner, we have only one class of isomorphic types $C(K)$, for $K \in \mathcal{L}$ (joint results with M. Korpalski).

11:15 – 11:35

Sebastian Jachimek

University of Wrocław

On the spaces dual to combinatorial Banach spaces

During the talk I will present quasi-Banach spaces which are closely related to the duals of combinatorial Banach spaces. More precisely, for a compact family $\mathcal{F}$ of finite subsets of $\mathbb{N}$ we define a quasi-norm whose Banach envelope is the dual norm for the combinatorial space generated by $\mathcal{F}$. Such quasi-norms seem to be much easier to handle than the dual norms and yet the quasi-Banach spaces induced by them share many properties with the dual spaces.

11:45 – 12:05

Víctor Olmos Prieto

UNED

The Banach–Saks rank of a separable weakly compact set

A well-known result by S. Banach and S. Saks states that every bounded sequence in $L^p([0,1])$, $1 < p < \infty$, has a Cesàro convergent subsequence. By taking the average of the subsequence multiple times, and through a recursive procedure, one can define a transfinite sequence of properties, thus giving rise to an ordinal rank or index for certain subsets of Banach spaces. We explore a different approach to this rank in the separable case using uniform families of finite subsets of integers. This allows us to characterize the existence of such a rank and study some of its properties from the point of view of Descriptive Set Theory.

12:15 – 12:35

Zdeněk Silber

Czech Academy of Sciences, Prague

A separable Banach space of nontrivial Baire order

We consider intrinsic Baire classes of a separable Banach space X. Those are defined recursively as iterated weak* sequential closures of the canonical image of X in X^{**}. We say that X is of Baire order alpha if alpha is the smallest ordinal in which this iteration stabilizes. As all the known examples of Baire orders of separable Banach spaces were 0, 1, or ω_1, this inspired Argyros, Godefroy and Rosenthal to ask whether there exists a separable Banach space of order 2. In the talk we show how to construct such space, solving this question positively. This is a joint work with Anna Pelczar-Barwacz and Tomasz Wawrzycki.

16:00 – 16:20

Maciej Korpalski

University of Wrocław

Semadeni derivative of Banach spaces and functions on nonmetrizable rectangles

We study Banach spaces $C(K)$ of real-valued continuous functions from the finite product of compact lines. It turns out that the topological character of these compact lines can be used to distinguish whether two spaces of continuous functions on products are isomorphic or embeddable to each other. In particular, for compact lines $K_1, \ldots, K_n, L_1, \ldots, L_k$ of uncountable character and $k \neq n$, we claim that Banach spaces $C(\prod_{i=1}^{n} K_i)$ and $C(\prod_{j=1}^{k} L_j)$ are not isomorphic.

16:25 – 16:45

Eugene Bilokopytov

University of Alberta, Edmonton

Von Neumann–Maharam problem for vector lattices

The classical Von Neumann–Maharam problem asks for a characterization of Boolean algebras which are isomorphic to measure algebras of finite measures. While open ended, this problem has motivated lots of research in Analysis and Set theory. One can take a topological approach to this problem: if a complete Boolean algebra admits a metrizable order continuous uniformly exhaustable locally solid topology, then it is a measure algebra; it is then left to find conditions for existence of such a topology. The goal of the talk is to lay out similar considerations in the context of vector lattices.

FRIDAY 21.03

9:00 – 9:20

Arkady Leiderman

Ben-Gurion University of the Negev, Beer-Sheva

On w^*-binormality of the dual space $C_k(X)'$

If E is a lcs (locally convex space) with its dual E', by $\beta(E', E)$ and $w^* = \sigma(E', E)$ we mean the strong and the weak* topology of E', respectively.

Definition. [1] For a lcs E we say that its dual E' is *w^*-binormal* if for every disjoint $\beta(E', E)$-closed $A \subset E'$ and w^*-closed $B \subset E'$ there exist disjoint $\beta(E', E)$-open $D \subset E'$ and w^*-open $C \subset E'$ such that $A \subset C$ and $B \subset D$.

For a Tychonoff space X by $C_k(X)$ we denote lcs of continuous real-valued functions on X with the compact-open topology. Δ-spaces have been defined and investigated in the recent papers [2] and [3].

Theorem 1. [1] Let X be a pseudocompact space and assume that the strong dual $C_k(X)'$ of $C_k(X)$ is w^*-binormal. Then X is a Δ-space.

We proved also that for a Corson compact space X the converse of Theorem 1 is true. A new property which formally is stronger than being a w^*-binormal has been defined.

Theorem 2. [1] Let X be a compact space. Then $C(X)'$ is effectively w^*-binormal if and only if X is an effectively Δ-space.

Examples of compact effectively Δ-spaces include all scattered Eberlein compact spaces and one-point compactification of Isbell–Mrówka spaces $\Psi(\mathcal{A})$. We will present some other results about the class of compact effectively Δ-spaces and pose open problems.

References

[1] J. Kąkol, O. Kurka, A. Leiderman, *On Asplund spaces $C_k(X)$ and w^*-binormality*, Results Math., **78:203** (2023), 19 pp.

[2] J. Kąkol, A. Leiderman, *A characterization of X for which spaces $C_p(X)$ are distinguished and its applications*, Proc. Amer. Math. Soc., series B, **8** (2021), 86–99.

[3] J. Kąkol, A. Leiderman, *Basic properties of X for which the space $C_p(X)$ is distinguished*, Proc. Amer. Math. Soc., series B, **8** (2021), 267–280.

9:30 – 9:50

Todor Manev

Sofia University

Continuous functions on Fedorchuk compacta

We study spaces of continuous functions on limits of inverse systems of compact spaces, where the bonding mappings are fully closed. A mapping between Hausdorff compacta is called fully closed if the intersection of the images of any two closed disjoint subsets is finite. We give a characterization of such systems in terms of a relation between the space of continuous functions on the limit and continuous functions on certain types of trees. When, moreover, the fibers of neighboring bonding mappings are metrizable, such systems are known as Fedorchuk compact spaces. The stated property allows us to obtain locally uniformly rotund renormings on the spaces of continuous functions on a certain subclass of Fedorchuk compacta.

10:00 – 10:20

Ondřej Kalenda

Charles University, Prague

Simpliciality of vector-valued function spaces

Generalizations of the Choquet theory of integral representation to the setting of vector-valued function spaces goes back to 1980s. However, a satisfactory theory of uniqueness of representing measures which would be analogous to the scalar case is still missing. I will present recent joint results with Jiří Spurný in this direction. In particular, it turns out that there are two natural directions of possible generalization — we call them weak simpliciality and vector simpliciality. In general these two notions are incomparable, but for function spaces containing constants vector simpliciality is strictly stronger. I will focus on similarities and differences from the scalar theory.

10:50 – 11:10

David Muñoz-Lahoz

UAM-ICMAT

AM-algebras

A Banach lattice X is said to be an AM-space if $\|x \vee y\| = \max\{\|x\|, \|y\|\}$ for every $x, y \in X_+$. A classical theorem of Kakutani establishes that AM-spaces are precisely the closed sublattices of $C(K)$ spaces. This result provides an intrinsic characterization of the closed sublattices of $C(K)$. However, since $C(K)$ naturally carries a Banach algebra structure as well, a natural question arises: Can one characterize the closed subspaces of $C(K)$ that are simultaneously sublattices and subalgebras? In this talk, we demonstrate that such a characterization is indeed possible by adding a simple algebraic constraint to the AM-space condition. Introducing this interplay between the AM-space and algebraic structures will require a new characterization of the AM-space property. (This is joint work with Pedro Tradacete.)

11:20 – 11:40

Luis David Reyes Sáenz

National Autonomous University of Mexico, Mexico City

Q-points, Q-measures, and L-orthogonality

The concept of L-orthogonality has been the focus of research in the past decades, partly due to its connection with the presence of copies of l_1. In recent years this line of research has found connections with research in set theory, through the use of special ultrafilters; mainly Q-points. In this talk we will present the concept of Q-measures, a generalization of Q-points, its connection with the research in L-orthogonality and some consistency and independence results. This is joint work with Antonio Avilés, Gonzalo Martínez Cervantes, and Alejandro Poveda.

11:50 – 12:10

Alberto Salguero-Alarcón

Complutense University of Madrid

Free objects in p-Banach lattices

There is quite a tradition in the study of free objects in functional analysis. In particular, free Banach lattices have found quite a number of interesting applications transferring properties of Banach spaces to properties of Banach lattices. In this talk, we address the existence of such objects in the realm of p-Banach spaces, where $0 < p < 1$, and we apply their properties to produce some examples of projective p-Banach lattices. This is an ongoing work with Pedro Tradacete (ICMAT) and Nazaret Trejo (UCM).

12:15 – 12:35

Ondřej Zindulka

Czech Technical University, Prague

Fractal measures in Polish groups and Banach spaces: cardinal invariants

Uniformity and covering of standard s-dimensional Hausdorff measures on the line received attention most notably from Fremlin, Shelah and Steprans, (recently) Elekes and Steprans, and others. We calculate uniformity and covering of Hausdorff, packing and Hewitt–Stomberg measures induced by a general gauge on Polish groups and in particular the Cantor set and Baer–Specker group. In particular it turns out that these invariants may differ for the two mentioned groups.