On a Theorem of Tignol for Defectless extensions and its converse *

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Abstract. Let (K, v) be a Henselian valued field of arbitrary rank. In 1990, Tignol proved that if $(K', v')/(K, v)$ is a finite separable defectless extension of degree a prime number, then the set $A_{K'/K} = \{v(Tr_{K'/K}(\alpha)) - v'(\alpha) | \alpha \in K', \alpha \neq 0\}$ has a minimum element. This paper extends Tignol's result to all finite separable extensions. Moreover a characterization of finite separable defectless extensions is given by showing that $(K', v')/(K, v)$ is a defectless extension if and only if the set $A_{K'/K}$ has a minimum element. Our proof also leads to a new proof of the well known result that each finite extension of a formally $\wp$-adic field (or more generally of a finitely ramified valued field) is defectless.

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1. Introduction

Throughout this paper, v is a Henselian valuation of arbitrary rank of a field K with residue field $R(K)$ and $\bar{v}$ is the unique prolongation of v to a fixed algebraic closure $\overline{K}$ of K. A finite extension $(K',v')/(K,v)$ (or briefly K'/K) will be called defectless if $[K':K]=ef$ where e and f are respectively the index of ramification and the residual degree of v'/v. This extension will be referred to as tame if (a) it is defectless; (b) the residue field of v' is a separable extension of the residue field of v; (c) the ramification index of v'/v is not divisible by the characteristic of the residue field of v.

Let $(K',v')\subseteq(\overline{K},\bar{v})$ be a finite extension of (K,v). Since (K,v) is Henselian, for any α in K' and σ in $Gal(\overline{K}/K)$, $\bar{v}\circ\sigma(\alpha)=\bar{v}(\alpha)$ and consequently $v(Tr_{K'/K}(\alpha))\geq v'(\alpha)$; here and elsewhere Tr stands for the trace. In 1990, Tignol proved that if $(K',v')/(K,v)$ is a finite separable extension of degree any prime number, then the set $A_{K'/K}$ defined by

$$A_{K'/K}=\{v(Tr_{K'/K}(\alpha))-v'(\alpha)|\alpha\in K',\alpha\neq 0\} \tag{1}$$

has a minimum element provided $(K',v')/(K,v)$ is a defectless extension (cf.[4, Prop. 2.5] or [5, Lemma 1.1]). He also proved that the smallest element of $A_{K'/K}$ is zero in case $(K',v')/(K,v)$ is a tame extension. In 2000, Khanduja [2] proved that the above result of Tignol in fact holds for all finite tame extensions and showed that a finite separable extension (K',v') of a Henselian valued field (K,v) is tame if and only if zero is the minimum element of $A_{K'/K}$. We have observed that if $(K',v')/(K,v)$ is any finite separable defectless extension, then the set $A_{K'/K}$ has a minimum element (see Lemma 2.2). This gives rise to the following natural question.

Let $(K',v')/(K,v)$ be a finite separable extension for which the set $A_{K'/K}$ has a minimum element. Is it true that (K',v') is a defectless extension of (K,v)?

In this paper, we prove that the answer to the above question is in the affirmative. In other words, it is proved that a finite separable extension (K',v') of (K,v) is defectless if and only if the set $A_{K'/K}$ has a minimum element. It will be shown that this characterization of defectless extensions quickly implies that every finite extension of a finitely ramified valued field is defectless, thereby providing a new proof of this well known result. Recall that a valued field (K, v) is said to be finitely ramified if the value group of v admits a least positive element λ and there is a prime number p and a natural number e such that $v(p) = e\lambda$; such a valued field has characteristic 0 and p is the characteristic of its residue field.

In the course of proof we use the notion of valuation basis. A set $\{x_1, ..., x_n\}$ of elements of an n-dimensional extension (K', v') of (K, v) is said to be a valuation basis of $(K', v')/(K, v)$ if for every choice of elements $a_i \in K$, we have $v'(\sum_{i=1}^{n} a_i x_i) = \min_i \{v'(a_i x_i)\}$. Note that a valuation basis of $(K', v')/(K, v)$ is linearly independent over K and hence is a basis of K'/K.

The main result of the present paper is the following:

Theorem 1.1. *Let v be a Henselian valuation of arbitrary rank of a field K. Let K'/K be a finite separable extension and v' be the prolongation of v to K'. Then the following statements are equivalent.*
(i) (K', v') is a defectless extension of (K, v).
(ii) $(K', v')/(K, v)$ has a valuation basis.
(iii) The set $A_{K'/K} = \{v(Tr_{K'/K}(\beta)) - v'(\beta) \mid \beta \in K',\ \beta \neq 0\}$ has a minimum element.

The following corollary will be deduced from the above theorem.

Corollary 1.2. Each finite extension of a finitely ramified Henselian valued field is defectless.

2. Some preliminary results

Let (K, v) and $(\overline{K}, \bar{v})$ be as in the preceding section. For any ξ in the valuation ring of $\bar{v}$, ξ^* will denote its $\bar{v}$-residue, i.e., the image of ξ under the canonical homomorphism from the valuation ring of $\bar{v}$ onto its residue field.

The result of the following lemma is well known. For the sake of completeness, we give its proof here.

Lemma 2.1. *Let (K', v') be a finite defectless extension of a Henselian valued field (K, v). Then it has a valuation basis.*

Proof. Let $G \subseteq G'$ and $R(K) \subseteq R(K')$ denote respectively the value groups and the residue fields of v and v'. Let e and f stand respectively for the index of G in G' and the degree of the extension $R(K')/R(K)$. Choose elements $x_1, ..., x_e$ in K' for which the cosets $G + v'(x_1), ..., G + v'(x_e)$ are all distinct. Choose $y_1, ..., y_f$ in the valuation ring of v' such that their v'-residues $y_1^*, ..., y_f^*$ are linearly independent over $R(K)$. Observe that the extension $(K', v')/(K, v)$ being defectless, has degree ef. Claim is that the set $\{x_i y_j,\ 1 \le i \le e,\ 1 \le j \le f\}$ is a valuation basis of $(K', v')/(K, v)$. Suppose that the claim is false. Then there exists an element $x = \sum_{j=1}^{f} \sum_{i=1}^{e} a_{ij} x_i y_j$ in K' with a_{ij} in K for which $v'(x) > \min_{i,j}\{v'(a_{ij} x_i y_j)\}$. If necessary after renaming, we may assume that $\min_{i,j}\{v'(a_{ij} x_i y_j)\} = v'(a_{11} x_1 y_1)$. The elements $y_1^*, ..., y_f^*$ being linearly independent over $R(K)$ are non-zero and hence $v'(y_j) = 0,\ 1 \le j \le f$. Thus we have

$$v'(\sum_{i=1}^{e} \sum_{j=1}^{f} a_{ij} x_i y_j) > \min_{i,j}\{v'(a_{ij} x_i y_j)\} = v'(a_{11} x_1). \tag{2}$$

Since $G + v'(x_1)$ is different from $G + v'(x_i)$ when $2 \le i \le e$, it follows from the equality in (2) that $v'(a_{ij} x_i y_j) > v'(a_{11} x_1)$ for $2 \le i \le e,\ 1 \le j \le f$; consequently $v'(\sum_{i=2}^{e} \sum_{j=1}^{f} a_{ij} x_i y_j) > v'(a_{11} x_1)$. Therefore (2) implies that

$$v'(\sum_{j=1}^{f} a_{1j} x_1 y_j) > v'(a_{11} x_1).$$

The above inequality shows that $\sum_{j=1}^{f} (\frac{a_{1j}}{a_{11}})^* y_j^* = 0^*$ which contradicts the linear independence of $y_1^*, ..., y_f^*$ over $R(K)$. This contradiction proves the lemma.

Lemma 2.2. *Suppose that a finite separable extension (K', v') of a Henselian valued field (K, v) has a valuation basis $w_1, ..., w_n$. Then the set $A_{K'/K}$ defined by (1) has smallest element equal to $\min_{1 \le i \le n} \{v(Tr_{K'/K}(w_i)) - v'(w_i)\}$.*

Proof. Let $\beta = \sum_{i=1}^{n} a_i w_i$ be any non-zero element of $K', a_i \in K$. Then

$$v'(\beta) = \min_i v'(a_i w_i) = v'(a_k w_k) \quad (say). \tag{3}$$

Using the triangle law, we have

$$v(Tr_{K'/K}(\beta)) \ge \min_i \{v(a_i Tr_{K'/K}(w_i))\} = v(a_j) + v(Tr_{K'/K}(w_j)) \quad (say). \tag{4}$$

It follows from (3) and (4) that

$$\begin{aligned} v(Tr_{K'/K}(\beta)) - v'(\beta) &\ge v(a_j) + v(Tr_{K'/K}(w_j)) - v'(a_k w_k) \\ &\ge v(a_j) + v(Tr_{K'/K}(w_j)) - v'(a_j w_j) \\ &= v(Tr_{K'/K}(w_j)) - v'(w_j). \end{aligned}$$

Thus we have shown that for any $\beta \neq 0$ in K', the inequality

$$v(Tr_{K'/K}(\beta)) - v'(\beta) \ge \min_{1 \le i \le n} \{v(Tr_{K'/K}(w_i)) - v'(w_i)\}$$

holds as desired.

As usual, an extension $(K', v')/(K, v)$ (or briefly K'/K when the underlying valuations are clear) will be called an immediate extension if v' and v have the same value group and the same residue field.

Lemma 2.3. *Let (K', v') be a finite separable extension of a Henselian valued field (K, v). Let L be an intermediate field such that K'/L is an immediate extension of* *degree strictly greater than one. Then the set $A_{K'/K}$ defined by (1) does not have any minimum element.*

Proof. To prove the lemma, it is clearly enough to show that for any given non-zero element ξ in K', there exists an element η in K' satisfying the following two conditions

$$v'(\eta) > v'(\xi), \quad Tr_{K'/K}(\eta) = Tr_{K'/K}(\xi). \tag{5}$$

We split the proof in two cases.

Case (i). Char $K = 0$. In this case there exists a generator θ of the extension K'/L with $Tr_{K'/L}(\theta) = 0$. Since K'/L is an immediate extension, on replacing θ by θ/a for a suitable element $a \in L$, we can assume that

$$v'(\theta) = 0 \text{ and } \theta^* = 1^*. \tag{6}$$

Let ξ be any non-zero element of K'. Using the fact that K'/L is an immediate extension, we can choose an element c belonging to L satisfying

$$(\xi/c)^* = -1^*. \tag{7}$$

We verify that (5) holds for the element η defined by $\eta = \xi + c\theta$. It follows from (6) and (7) that

$$(\eta/\xi)^* = 1^* + (c/\xi)^*\theta^* = 0^*.$$

Therefore $v'(\eta) > v'(\xi)$. Since $Tr_{K'/L}(\theta) = 0$, we have

$$Tr_{K'/K}(\eta) = Tr_{K'/K}(\xi) + Tr_{L/K}(cTr_{K'/L}(\theta)) = Tr_{K'/K}(\xi)$$

as desired.

Case (ii). Char $K = p > 0$. Let ξ be any non-zero element of K'. Fix an element c of L satisfying (7). Define an element η of K' by $\eta = \xi + c$. Then clearly

$$(\eta/\xi)^* = 1 + (c/\xi)^* = 0^*.$$

Since char $K = p > 0$, and K'/L is an extension of degree $p^r > 1$, we have $Tr_{K'/L}(c) = p^r c = 0$. Therefore η satisfies (5).

Lemma 2.4. *Let $(K', v')/(K, v)$ be a finite separable extension of Henselian valued fields. Let L be an intermediate field such that K'/L is a defectless extension with respect to the valuation obtained by restricting v' to L. Suppose that $A_{K'/K}$ has a minimum element, then $A_{L/K}$ has a minimum element.*

Proof. As K'/L is a defectless extension, it has a valuation basis $\theta_1, ..., \theta_m$ by virtue of Lemma 2.1. We denote $\min A_{K'/K}$ by λ and set

$$t_i = Tr_{K'/L}(\theta_i),\ 1 \leq i \leq m.$$

Let $\beta = \sum_{i=1}^{m} a_i\theta_i, a_i \in L$, be an element of K' such that $\lambda = v(Tr_{K'/K}(\beta)) - v'(\beta)$, i.e.,

$$\lambda = v(\sum_i Tr_{L/K}(a_i t_i)) - v'(\sum_i a_i\theta_i). \tag{8}$$

If an index s is defined so as

$$\min_i\{v(Tr_{L/K}(a_i t_i))\} = v(Tr_{L/K}(a_s t_s)), \tag{9}$$

then we are going to show that $a_s t_s \neq 0$ and

$$\lambda = v(Tr_{K'/K}(a_s\theta_s)) - v'(a_s\theta_s); \tag{10}$$

this will be used to prove that

$$\min A_{L/K} = v(Tr_{L/K}(a_s t_s)) - v'(a_s t_s) \tag{11}$$

which will complete the proof of the lemma.

Observe that $a_s t_s \neq 0$, for otherwise $Tr_{L/K}(a_i t_i) = 0$ for $1 \leq i \leq m$ by virtue of (9); this would imply that $Tr_{K'/K}(\beta) = \sum_i Tr_{K'/K}(a_i\theta_i) = \sum_i Tr_{L/K}(a_i t_i) = 0$ leading to $\lambda = \infty$, which is impossible as K'/K is a separable extension. Using (8) and (9) and the fact that $\theta_1, ..., \theta_m$ is a valuation basis of K'/L, we see that

$$\lambda \geq \min_i\{v(Tr_{L/K}(a_i t_i)\} - \min_i\{v'(a_i\theta_i)\}$$

$$\geq v(Tr_{L/K}(a_s t_s)) - v'(a_s\theta_s)$$

$$= v(Tr_{K'/K}(a_s\theta_s)) - v'(a_s\theta_s).$$

Indeed the inequality $\lambda \geq v(Tr_{K'/K}(a_s\theta_s)) - v'(a_s\theta_s)$ just proved must be an equality by virtue of the fact that λ is minimum of $A_{K'/K}$. This proves (10).

Suppose to the contrary that (11) is false. Then there exists a non-zero element c of L such that

$$v(Tr_{L/K}(c)) - v'(c) < v(Tr_{L/K}(a_s t_s)) - v'(a_s t_s). \tag{12}$$

As $t_s \neq 0$, we can write c as $bt_s, b \in L$. Consider the element $b\theta_s$ of K'. Keeping in mind (12) and the equality $Tr_{K'/L}(\theta_s) = t_s$, a simple calculation shows that

$$v(Tr_{K'/K}(b\theta_s)) - v'(b\theta_s) = v(Tr_{L/K}(bt_s)) - v'(b\theta_s)$$

$$< v(Tr_{L/K}(a_s t_s)) - v'(a_s t_s) + v'(bt_s) - v'(b\theta_s)$$

$$= v(Tr_{K'/K}(a_s\theta_s)) - v'(a_s\theta_s).$$

Therefore it now follows from (10) that

$$v(Tr_{K'/K}(b\theta_s)) - v'(b\theta_s) < \lambda$$

which is impossible as λ is the minimum element of the set $A_{K'/K}$. This contradiction proves (11) and hence the lemma.

We shall use the following already known theorem. Its proof is omitted (see [2]).

Theorem 2.A. *A finite separable extension (K', v') of a Henselian valued field (K, v) is tame if and only if there exists $\alpha \neq 0$ in K' satisfying $v(Tr_{K'/K}(\alpha)) = v'(\alpha)$.*

We now prove a theorem which will be used to prove Theorem 1.1; it is of independent interest as well.

Theorem 2.5. *Let $(K,v) \subset (K',v') \subset (K'',v'')$ be a tower of finite separable extensions. Suppose that $A_{K''/K'}$ and $A_{K'/K}$ have minimum elements. Then $A_{K''/K}$ has a minimum element which equals* $\min A_{K''/K'} + \min A_{K'/K}$.

Proof. Let α be any non-zero element of K''. We can write

$$v(Tr_{K''/K}(\alpha)) - v''(\alpha) = v(Tr_{K'/K}(Tr_{K''/K'}(\alpha))) - v'(Tr_{K''/K'}(\alpha)) +$$

$$v'(Tr_{K''/K'}(\alpha)) - v''(\alpha).$$

This shows that $A_{K''/K} \subset A_{K''/K'} + A_{K'/K}$; hence $A_{K''/K}$ is bounded from below by $\min A_{K''/K'} + \min A_{K'/K}$. On the other hand, if $a' \in K'$ and $\gamma \in K''$ satisfy

$$v(Tr_{K'/K}(a')) - v'(a') = \min A_{K'/K}$$

and

$$v'(Tr_{K''/K'}(\gamma)) - v''(\gamma) = \min A_{K''/K'},$$

then one can quickly verify that $b = \gamma a' Tr_{K''/K'}(\gamma)^{-1}$ satisfies

$$v(Tr_{K''/K}(b)) - v''(b) = \min A_{K''/K'} + \min A_{K'/K};$$

hence $\min A_{K''/K'} + \min A_{K'/K} \in A_{K''/K}$. The theorem follows.

The corollary stated below is an immediate consequence of the above theorem and Theorem 2.A.

Corollary. *Let $(K,v) \subseteq (K',v') \subseteq (K'',v'')$ be a tower of finite separable extensions such that K''/K' is a tame extension. Suppose that $A_{K'/K}$ has a minimum element. Then $A_{K''/K}$ has a minimum element which equals* $\min A_{K'/K}$.

The following theorem which will be used in the sequel is essentially proved in [3, Lemma 3.15]. For the sake of readers' convenience and ready reference, we give its proof here.

Theorem 2.6. *Let v be a Henselian valuation of a field K whose residue field is of characteristic $p > 0$. Let w be its prolongation to the separable closure K^{sep} of K. Let $K' \subseteq K^{sep}$ be a finite extension of K which is not tame. Then there exists a finite tame extension T of K such that TK'/T is a tower of extensions of degree p each.*

Proof. Let K^V denote the maximal tame extension of (K, v) contained in (K^{sep}, w). By ramification theory, K^V is the ramification field of the extension $(K^{sep}, w)/(K, v)$ and K^{sep}/K^V is a p-extension (cf. [1, 22.7, 20.18]). Write $K' = K(\alpha)$. Let $K^V(\alpha_1, ..., \alpha_s)$ be the smallest Galois extension of K^V containing α. Consider the groups

$$H_o = Gal(K^V(\alpha_1, ..., \alpha_s)/K^V), \quad H = Gal(K^V(\alpha_1, ..., \alpha_s)/K^V(\alpha)).$$

Since K'/K is not a tame extension, α does not belong to K^V. Therefore $\mid H_o \mid > 1$; in fact by what has been said in the above paragraph, the order of H_o must be a power of p. So there exists a descending chain of subgroups

$$H_o \supset H_1 \supset ... \supset H_t = H \supset H_{t+1} \supset ... \supset \{e\}$$

such that each H_i is a normal subgroup of H_{i-1} of index p. Let $K^V(\beta_1), K^V(\beta_1, \beta_2), ...,$ $K^V(\beta_1, ..., \beta_t) = K^V(\alpha)$ denote respectively the fixed fields of $H_1, ..., H_t = H$. It is clear that

$$K^V \subset K^V(\beta_1) \subset K^V(\beta_1, \beta_2) \subset ... \subset K^V(\beta_1, ..., \beta_t) = K^V(\alpha) \tag{13}$$

is a tower of extensions of degree p each. Assume without loss of generality that $\beta_t = \alpha$. Let $X^p + a_{11}X^{p-1} + ... + a_{1p}$ be the minimal polynomial of β_1 over K^V. Let K_1 denote the field obtained by adjoining to K the coefficients $a_{11}, ..., a_{1p}$. Let $X^p + b_{21}X^{p-1} + ... + b_{2p}$ be the minimal polynomial of β_2 over $K^V[\beta_1]$. We can write b_{2i} as

$$b_{2i} = \sum_{j=0}^{p-1} a_{2ij}\beta_1^j, \ a_{2ij} \in K^V.$$

Let K_2 denote the field obtained by adjoining to K_1 the p^2 elements $\{a_{2ij}, \ 1 \leq i \leq p, \ 0 \leq j \leq p-1\}$. Repeating this process t times, we obtain a subfield K_t of K^V which is a finite tame extension of K. Denote K_t by T. Clearly

$$T \subset T(\beta_1) \subset T(\beta_1, \beta_2) \subset ... \subset T(\beta_1, ..., \beta_t) \tag{14}$$

is a tower of extensions of degree p each. Since $T(\beta_1, ..., \beta_t)$ contains $\beta_t = \alpha$ and α is algebraic over K^V of degree p^t by virtue of (13), it now follows from (14) that $T(\beta_1, ..., \beta_t) = T(\alpha) = TK'$. This completes the proof of the theorem.

3. **Proof of Theorem 1.1 and Corollary 1.2.**

Proof of Theorem 1.1. The assertions $(i) \implies (ii)$ and $(ii) \implies (iii)$ hold in view of Lemma 2.1 and Lemma 2.2 respectively. We now prove $(iii) \implies (i)$. Since every finite tame extension is defectless, it may be assumed that K'/K is not a tame extension. Let the prime number p denote the characteristic of the residue field of v. Applying Theorem 2.6, we see that there exists a tame extension T of K such that TK'/T is a tower of extensions $T \subset T_1 \subset ... \subset T_s = TK'$ of degree p each. Since tameness is preserved under composition [1, 20.15(b)], TK'/K' is a tame extension. By hypothesis $A_{K'/K}$ has a minimum element. Therefore by the corollory following Theorem 2.5, $\min A_{TK'/K}$ exists. It now follows from Lemma 2.3 that the extension $T_s = TK'$ of T_{s-1} having degree p is defectless. Now applying Lemma 2.4 to the tower of extensions $K \subset T_{s-1} \subset T_s$, we see that $\min A_{T_{s-1}/K}$ exists. Repetition of the above argument (with T_s replaced by T_{s-1}) yields that T_{s-2}/T_{s-1} is defectless and $\min A_{T_{s-2}/K}$ exists. Continuing this process s times, we conclude that $T_s = TK'$ is a defectless extension of T. Also T/K being tame is defectless. Consequently TK'/K is a defectless extension and so is K'/K.

Proof of Corollary 1.2. Let (K', v') be an extension of a finitely ramified Henselian valued field (K, v) of degree n. Let p be the characteristic of the residue field of v and $v(p)/e$ be the least positive element of the value group G of v. Let r be the largest positive integer such that $v(p)/er$ belongs to the value group G' of v'. We indeed verify that the smallest convex subgroup C of G' containing $v(p)$ is the cyclic group generated by

$v(p)/er$. Note that an element g' of G' belongs to C if and only if $\max\{g', -g'\} \leq sv(p)$ for some positive integer s. Let h be any positive element of C. There exists a non-negative integer m such that $mv(p)/e \leq nh < (m+1)v(p)/e$. As $v(p)/e$ is the least positive element of G and $nh - mv(p)/e$ belongs to G, it follows that $nh = mv(p)/e$. So we can write $h = av(p)/ber$ where a and b are coprime positive integers. If a', b' are integers satisfying $aa' + bb' = 1$, then it is clear that $v(p)/ber = a'h + (b'v(p)/er)$ is an element of G'. Since r is the largest integer such that $v(p)/er$ belongs to G', we conclude that $b = 1$ and hence $h = av(p)/er$ is in the cyclic group generated by $v(p)/er$ as desired.

To prove that $(K', v')/(K, v)$ is defectless, in view of Theorem 1.1, it is enough to show that the set $A_{K'/K}$ has a minimum element. Observe that $v(Tr_{K'/K}(1)) - v(1) = v(n)$ belongs to $A_{K'/K} \bigcap C$. Since C is the cyclic group generated by $v(p)/er$, it follows that $\min A_{K'/K} = qv(p)/er$ where q is the least non-negative integer such that $qv(p)/er$ belongs to $A_{K'/K}$.

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